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Social optimum in the Basic Climate Economy (BCE) model

The Basic Climate Economy (BCE) model is a useful tool which is able to deal with climate economics and climate policy, incorporating non-renewable (fossil) stock depletion, pollution stock accumulation, endogenous growth and climate-induced capital depreciation. Specifically, the BCE model adds stock pollution to a two-sector capital-resource model; we have the manufacturing sector which produces goods immediately available to consumption, and the corporate sector that provides goods and services for investments which increase the physical capital stock.

Here we can make the following modeling assumptions: constant returns to capital in capital sector, natural resource use for final goods, emissions (caused by resource use) add to pollution stock, and capital depreciation is increased by pollution stock.

.1 Assumptions

For the *production*, the final output Y_t is produced by capital K_t and natural resources R_t according to the Cobb-Douglas form:

$$Y_t = A\left(\epsilon_t K_t\right)^{\alpha} R_t^{1-\alpha} \tag{1}$$

where $\epsilon_t \equiv \frac{K_{Y_t}}{K_t} \in [0,1]$ is the aggregate fraction of capital devoted to the consumption good.

For the *resources*, let R be a natural resource which is extracted from resource stock S and regenerated by nature at a rate ζ . We have the following motion equation:

$$\dot{S}_t = \zeta S_t - R_t \tag{2}$$

Here we assume the resource R to be exhaustible, $\zeta = 0$. Hence,

$$\dot{S}_t = -R_t \quad s.t. \quad \int_0^{\infty} R_t dt \leq S_0 \tag{3}$$

For the *capital*, capital K_t is assumed to be the only input, so that investment good I_t can be written as:

$$I_t = B\left(1-\epsilon_t\right) K_t \quad with \quad B > 0 \tag{4}$$

Investment leads to capital accumulation according to

$$\dot{K}_t = I_t - D_t\left(P_t\right) K_t = \; = B\left(1-\epsilon_t\right) K_t - D_t\left(P_t\right) K_t \tag{5}$$

where $D_t\left(P_t\right)$ is the damage function. Given an endogenous rate of capital depreciation, $D_t\left(P_t\right) \in [0,1]$ denotes the share of capital loss.

For the *pollution*, pollution stock P increases with the use of natural resources and could decay at a rate ω:

$$\dot{P}_t = \phi R_t - \omega P_t \quad with \quad \phi > 0, \omega \in [0,1] \tag{6}$$

where ϕ represents the carbon intensity of the polluting resource. Here we assume no decay of pollution stock, i.e. $\omega = 0$. Hence,

$$\dot{P}_t = \phi R_t = -\phi \dot{S}_t \tag{7}$$

Moreover, we assume there is no emission mitigation technology available. Finally, we can suppose that the stock of exhaustible resources and the state of the atmosphere influence individual utility, together with (negatively) affecting the production and the capital stock.

.2 Social optimum

Given all the assumptions above, the social planner chooses the share ϵ_t, and resource extraction R_t in order to maximize the utility function, which will have Equations 1,4,5,3 as constraints. The utility function $U(C_t)$ is given by a CRRA-type function:

$$\max_{C_t,\epsilon_t} \int_0^\infty U(C_t)\, e^{-\rho t} dt = \max_{C_t,\epsilon_t} \int_0^\infty \frac{C_t^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt \tag{8}$$

with with $\sigma > 0$, the inverse of the elasticity of intertemporal substitution.

We assume to have an equilibrium condition on consumer goods market, which ensures $Y_t = C_t$ and the no-arbitrage condition on the prices $p_Y = p_C$ (and on the shadow prices $\lambda_{C_t} = \lambda_{Y_t}$). Hence,

$$C_t = Y_t = A(\epsilon_t K_t)^\alpha R_t^{1-\alpha} \tag{9}$$

In order to optimize the problem, we need to write the expression for the current-value Hamiltonian H_t, taking into account the CRRA utility function and all the constraints.

$$H_t = \frac{C_t^{1-\sigma}-1}{1-\sigma} + \lambda_{C_t}\left[A(\epsilon_t K_t)^\alpha R_t^{1-\alpha} - C_t\right] + \lambda_{K_t} K_t \left[B(1-\epsilon_t) - D(P_t)\right] - \lambda_{S_t} R_t + \lambda_{P_t} \phi R_t \tag{10}$$

where λ_{C_t},λ_{K_t},λ_{S_t}, λ_{C_t}, λ_{P_t} are the shadow prices of the consumption good, C_t, capital stock, K_t, stock of non-renewable resources, S_t, and stock of pollution, λ_{P_t}. Assuming an internal solution, we are able to write the first order conditions $\left(\frac{\partial H_t}{\partial(\cdot)} = 0\right)$ with reference to C_t, ϵ_t and R_t:

$$\frac{\partial H_t}{\partial C_t} = 0 \quad \Rightarrow \quad \frac{1-\sigma}{1-\sigma} C^{-\sigma} - \lambda_{C_t} = 0; \quad C_t^{-\sigma} - \lambda_{C_t} = 0 \tag{11}$$

$$\frac{\partial H_t}{\partial \epsilon_t} = 0 \quad \Rightarrow \quad \lambda_{C_t} \alpha A K_t^\alpha R_t^{1-\alpha} \epsilon_t^{\alpha-1} - \lambda_{K_t} B K_t = 0 \tag{12}$$

$$\frac{\partial H_t}{\partial R_t} = 0 \quad \Rightarrow \quad \lambda_{C_t}(1-\alpha) A(\epsilon_t K_t)^\alpha R_t^{-\alpha} - \lambda_{S_t} + \lambda_{P_t}\phi = 0 \tag{13}$$

Rearranging Equations 11,12,13, we obtain:

$$C_t^{-\sigma} = \lambda_{C_t} \tag{14}$$

$$\lambda_{C_t} \alpha A (\epsilon_t K_t)^\alpha R_t^{1-\alpha} = \lambda_{K_t} B K_t \epsilon_t \quad \xrightarrow{Eqn.9} \quad \lambda_{C_t} \alpha C_t = \lambda_{K_t} B K_t \epsilon_t \quad \to \quad \alpha \frac{C_t}{K_t} = \frac{\lambda_{K_t}}{\lambda_{C_t}} B \epsilon_t \tag{15}$$

$$\lambda_{C_t}(1-\alpha) A(\epsilon_t K_t)^\alpha R_t^{-\alpha} \frac{R_t}{R_t} = \lambda_{S_t} - \lambda_{P_t}\phi \quad \xrightarrow{Eqn.9} \quad \lambda_{C_t}(1-\alpha) C_t \frac{1}{R_t} = \lambda_{S_t} - \lambda_{P_t}\phi \quad \xrightarrow{continues} \tag{16}$$

$$\xrightarrow{continues} \quad (1-\alpha)\frac{C_t}{R_t} = \frac{\lambda_{S_t}}{\lambda_{C_t}} - \frac{\lambda_{P_t}}{\lambda_{C_t}}\phi; \quad (1-\alpha)\frac{C_t}{R_t} = \frac{1}{\lambda_{C_t}}(\lambda_{S_t} - \phi\lambda_{P_t}) \tag{17}$$

Moreover, we know that $\frac{\partial H_t}{\partial (*)} = \rho \lambda_{*_t} - \dot{\lambda}_{*_t}$ for every state variable, here K_t, S_t and P_t, with its shadow price λ_{*_t}. Therefore, we can write:

$$\frac{\partial H_t}{\partial K_t} = \rho\lambda_{K_t} - \dot{\lambda}_{K_t} \quad \Rightarrow \quad \lambda_{C_t} \alpha A \epsilon^\alpha K_t^{\alpha-1} R_t^{1-\alpha} + \lambda_{K_t}\left[B(1-\epsilon_t) - D(P_t)\right] = \rho\lambda_{K_t} - \dot{\lambda}_{K_t} \tag{18}$$

$$\frac{\partial H_t}{\partial S_t} = \rho\lambda_{S_t} - \dot{\lambda}_{S_t} \quad \Rightarrow \quad 0 = \rho\lambda_{S_t} - \dot{\lambda}_{S_t} \tag{19}$$

$$\frac{\partial H_t}{\partial P_t} = \rho\lambda_{P_t} - \dot{\lambda}_{P_t} \quad \Rightarrow \quad -\lambda_{K_t} K_t \dot{D}(P_t) = \rho\lambda_{P_t} - \dot{\lambda}_{P_t} \tag{20}$$

Rewriting Equations 18 we obtain:

$$\lambda_{C_t} \alpha A \epsilon^{\alpha} K_t^{\alpha} R_t^{1-\alpha} \frac{1}{K_t} + \lambda_{K_t} \left[B\left(1-\epsilon_t\right) - D\left(P_t\right)\right] \frac{K_t}{K_t} = \rho \lambda_{K_t} - \dot{\lambda}_{K_t} \tag{21}$$

$$\xrightarrow{\textit{Equations 5 and 9}} \quad \lambda_{C_t} \alpha C_t \frac{1}{K_t} + \lambda_{K_t} \frac{\dot{K}_t}{K_t} = \rho \lambda_{K_t} - \dot{\lambda}_{K_t} \tag{22}$$

$$\xrightarrow{\textit{Dividing by } \lambda_{K_t}} \quad \alpha \frac{\lambda_{C_t} C_t}{\lambda_{K_t} K_t} + \frac{\dot{K}_t}{K_t} = \rho - \frac{\dot{\lambda}_{K_t}}{\lambda_{K_t}} \tag{23}$$

$$\xrightarrow{\textit{Rearranging}} \quad \frac{\dot{\lambda}_{K_t}}{\lambda_{K_t}} + \frac{\dot{K}_t}{K_t} = -\alpha \frac{\lambda_{C_t} C_t}{\lambda_{K_t} K_t} + \rho; \quad \frac{\dot{\lambda}_{K_t} K_t + \dot{K}_t \lambda_{K_t}}{\lambda_{K_t} K_t} = -\alpha \frac{\lambda_{C_t} C_t}{\lambda_{K_t} K_t} + \rho \tag{24}$$

$$\xrightarrow{\textit{We identify the growth rates } \frac{\dot{\circ}}{\circ} = \hat{\circ}} \quad \widehat{\lambda_{K_t} K_t} = -\alpha \frac{\lambda_{C_t} C_t}{\lambda_{K_t} K_t} + \rho \tag{25}$$

Rewriting Equations 19 we obtain:

$$\hat{\lambda}_{S_t} = \rho \tag{26}$$

Rewriting Equations 20 we obtain:

$$\dot{\lambda}_{P_t} = \lambda_{K_t} K_t \dot{D}(P_t) + \rho \lambda_{P_t} \tag{27}$$

$$\xrightarrow{\textit{Dividing by } \lambda_{P_t}} \quad \hat{\lambda}_{P_t} = \dot{D}(P_t) K_t \frac{\lambda_{K_t}}{\lambda_{P_t}} + \rho \tag{28}$$

The optimization must also include appropriate transversality conditions:

$$\lim_{t\to\infty} \lambda_{S_t} S_t e^{-\rho t} = 0 \tag{29}$$

$$\lim_{t\to\infty} \lambda_{K_t} K_t e^{-\rho t} = 0 \tag{30}$$

$$\lim_{t\to\infty} \lambda_{P_t} P_t e^{-\rho t} = 0 \tag{31}$$

We can notice in Equation 26 the **Hotelling rule** for the extraction of the non-renewable resource.

Moreover, Equation 15 suggests that it is indifferent allocating capital between the two activities of the BCE model: producing the investment good and the consumption good. If we had $\sigma = 1$ (i.e logarithmic utility), we would have from Equation 14:

$$\lambda_{C_t} C_t = 1 \quad \xrightarrow{\textit{Eqn.15}} \quad \lambda_{K_t} K_t = \frac{\alpha}{B\epsilon_t} \quad \xrightarrow{\textit{Eqn.25}} \quad \frac{d\left(\lambda_{K_t} K_t\right)}{\lambda_{K_t} K_t} = -B\epsilon + \rho; \quad \dot{\epsilon}_t = B{\epsilon_t}^2 - \rho \epsilon_t \tag{32}$$

$$\xrightarrow{\textit{Solving the differential equation}} \quad \epsilon_t = \frac{\rho C_1 e^{-\rho t}}{B C_1 e^{-\rho t} + B \rho C_2} \tag{33}$$

Using the transversality condition (30):

$$\lim_{t\to\infty} \frac{\alpha}{B\epsilon_t} e^{-\rho t} = 0 \quad \xrightarrow{\textit{Eqn.33}} \quad \lim_{t\to\infty} \frac{\alpha}{\rho C_1} \left(C_1 e^{-\rho t} + \rho C_2\right) = 0 \quad \Rightarrow \quad C_2 = 0 \tag{34}$$

Therefore we derive that:

$$\epsilon = \frac{\rho}{B} \equiv \bar{\epsilon} \tag{35}$$

Equation 35 indicates that the share of capital used in the final goods sector instantaneously jumps to its *steady-state value*.

In order to find the Keynes-Ramsey rule (KRR) for the optimal growth rate of consumption $\hat{C}_t$, we consider Equation 9:

$$C_t = A\left(\epsilon_t K_t\right)^{\alpha} R_t^{1-\alpha} \xrightarrow{\textit{Natural logarithm}} \ln C_t = \ln A + \alpha \ln \epsilon_t + \alpha \ln K_t + (1-\alpha)\ln R_t \tag{36}$$

$$\xrightarrow[\textit{Recalling the growth rates } \frac{\dot{\circ}}{\circ}=\hat{\circ}]{\textit{Taking the derivative}} \hat{C}_t = \alpha\hat{\epsilon}_t + \alpha\hat{K}_t + (1-\alpha)\hat{R}_t \tag{37}$$

Recalling Equation 5:

$$\dot{K}_t = B\left(1-\epsilon_t\right)K_t - D_t\left(P_t\right)K_t \xrightarrow{\textit{Dividing by } K_t} \hat{K}_t = B\left(1-\epsilon_t\right) - D_t\left(P_t\right) \tag{38}$$

Recalling Equation 14:

$$C_t^{-\sigma} = \lambda_{C_t} \xrightarrow{\textit{Natural logarithm}} -\sigma \ln C_t = \ln \lambda_{C_t} \xrightarrow{\textit{Derivative}} \hat{\lambda}_{C_t} = -\sigma\hat{C}_t \tag{39}$$

From Equation 15:

$$(1-\alpha)\frac{C_t}{R_t} = \frac{1}{\lambda_{C_t}}\left(\lambda_{S_t} - \lambda_{P_t}\right)\phi \xrightarrow{\textit{Nat. log.}} \ln(1-\alpha) + \ln C_t - \ln R_t = -\ln\lambda_{C_t} + \ln\left(\lambda_{S_t} - \lambda_{P_t}\right) + \ln\phi \tag{40}$$

$$\xrightarrow{\textit{Deriv.}} \hat{C}_t - \hat{R}_t = -\hat{\lambda}_{C_t} + \frac{\dot{\lambda}_{S_t} - \dot{\lambda}_{P_t}}{\lambda_{S_t} - \lambda_{P_t}};\ \hat{R}_t = \hat{C}_t + \hat{\lambda}_{C_t} - \frac{\dot{\lambda}_{S_t}}{\lambda_{S_t}}\left(\frac{1-\frac{\dot{\lambda}_{P_t}}{\dot{\lambda}_{S_t}}}{1-\frac{\lambda_{P_t}}{\lambda_{S_t}}}\right);\ \hat{R}_t = \hat{C}_t + \hat{\lambda}_{C_t} - \hat{\lambda}_{S_t}\left(\frac{1-\frac{\dot{\lambda}_{P_t}}{\dot{\lambda}_{S_t}}}{1-\frac{\lambda_{P_t}}{\lambda_{S_t}}}\right) \tag{41}$$

$$\xrightarrow{\textit{Equations 26 and 38}} \hat{R}_t = \hat{C}_t - \sigma\hat{C}_t - \rho\left(\frac{1-\frac{\dot{\lambda}_{P_t}}{\dot{\lambda}_{S_t}}}{1-\frac{\lambda_{P_t}}{\lambda_{S_t}}}\right);\quad \hat{R}_t = (1-\sigma)\hat{C}_t - \rho\left(\frac{1-\frac{\dot{\lambda}_{P_t}}{\dot{\lambda}_{S_t}}}{1-\frac{\lambda_{P_t}}{\lambda_{S_t}}}\right) \tag{42}$$

From Equation 17:

$$\alpha\frac{C_t}{K_t} = \frac{\lambda_{K_t}}{\lambda_{C_t}}B\epsilon_t \xrightarrow{\textit{Nat. log.}} \ln(\alpha) + \ln C_t - \ln K_t = \ln\lambda_{K_t} - \ln\lambda_{C_t} + \ln B + \ln\epsilon_t \tag{43}$$

$$\xrightarrow{\textit{Deriv.}} \hat{C}_t - \hat{K}_t = \hat{\lambda}_{K_t} - \hat{\lambda}_{C_t} + \hat{\epsilon}_t \xrightarrow[\textit{Equations 38 and 39}]{\textit{Rearranging}} \hat{\epsilon}_t = \hat{C}_t - \left(B\left(1-\epsilon_t\right) - D_t\left(P_t\right)\right) - \hat{\lambda}_{K_t} - \sigma\hat{C}_t \tag{44}$$

From Equation 25:

$$\hat{\lambda}_{K_t} + \hat{K}_t = -\alpha\frac{\lambda_{C_t}C_t}{\lambda_{K_t}K_t} + \rho \xrightarrow{\textit{Eq 15}} \hat{\lambda}_{K_t} + \hat{K}_t = -B\epsilon_t + \rho;\ \xrightarrow{\textit{Eq 38}} \hat{\lambda}_{K_t} = -B\epsilon_t + \rho - \left(B\left(1-\epsilon_t\right) - D_t\left(P_t\right)\right) \tag{45}$$

Substituting Equation 45 into Equation 44:

$$\hat{\epsilon}_t = \hat{C}_t - \left(B\left(1-\epsilon_t\right) - D_t\left(P_t\right)\right) - \left(-B\epsilon_t + \rho - \left(B\left(1-\epsilon_t\right) - D_t\left(P_t\right)\right)\right) - \sigma\hat{C}_t \tag{46}$$

$$\hat{\epsilon}_t = B\epsilon_t + (1-\sigma)\hat{C}_t - \rho \tag{47}$$

Finally, we can substitute Equation 38, 42, 47 into Equation 37:

$$\hat{C}_t = \alpha\left[B\epsilon_t + (1-\sigma)\hat{C}_t - \rho\right] + \alpha\left[B\left(1-\epsilon_t\right) - D_t\left(P_t\right)\right] + (1-\alpha)\left[(1-\sigma)\hat{C}_t - \rho\left(\frac{1-\frac{\dot{\lambda}_{P_t}}{\dot{\lambda}_{S_t}}}{1-\frac{\lambda_{P_t}}{\lambda_{S_t}}}\right)\right] \tag{48}$$

$$\hat{C}_t = \alpha B\epsilon_t + \alpha(1-\sigma)\hat{C}_t - \alpha\rho + \alpha B - \alpha B\epsilon_t - \alpha D_t\left(P_t\right) + (1-\sigma)\hat{C}_t - \alpha(1-\sigma)\hat{C}_t - (1-\alpha)\rho\left(\frac{1-\frac{\dot{\lambda}_{P_t}}{\dot{\lambda}_{S_t}}}{1-\frac{\lambda_{P_t}}{\lambda_{S_t}}}\right) \tag{49}$$

$$\sigma\hat{C}_t = -\alpha\rho + \alpha B - \alpha D_t\left(P_t\right) - (1-\alpha)\rho\left(\frac{1-\frac{\dot{\lambda}_{P_t}}{\dot{\lambda}_{S_t}}}{1-\frac{\lambda_{P_t}}{\lambda_{S_t}}}\right) \tag{50}$$

$$\hat{C}_t = \frac{1}{\sigma}\left[\alpha B - \alpha D_t\left(P_t\right) - \alpha\rho - \rho\left(\alpha + \alpha\left(\frac{1-\frac{\dot{\lambda}_{P_t}}{\dot{\lambda}_{S_t}}}{1-\frac{\lambda_{P_t}}{\lambda_{S_t}}}\right) + \left(\frac{1-\frac{\dot{\lambda}_{P_t}}{\dot{\lambda}_{S_t}}}{1-\frac{\lambda_{P_t}}{\lambda_{S_t}}}\right)\right)\right] \tag{51}$$

We can notice that Equation 51 is the **Keynes-Ramsey rule** for the optimal growth rate of consumption.

In particular, if we can assume that the stock of exhaustible resources and the state of the atmosphere do NOT primarily affect individual utility, we will have the shadow price for the stock of pollution equal to zero:

$$\lambda_{P_t} = 0 \quad therefore \quad \dot{\lambda}_{P_t} = 0 \tag{52}$$

and the Keynes-Ramsey rule as written in Equation 51 is reduced to

$$\hat{C}_t = \frac{\alpha B}{\sigma} - \frac{\alpha D\left(P_t\right)}{\sigma} - \frac{\rho}{\sigma} \tag{53}$$

Equation 53 coincide with the KRR obtained with the decentralized equilibrium in the BCE model paper [Bretschger, L., Karydas, C.,(2018)]. We can notice in Equation 53 the different aspects of productivity, depreciation, and discounting on the growth rate of consumption in time.

In particular, for any given damage function $D\left(P_t\right)$, the dynamic system expressed by Equations 47 and 53, along with the resource and climate dynamics (3), (7), and the transversality conditions (29), (30), (31), are sufficient to completely characterize the social optimum model.

The **steady-state values** that we can immediately derive from the above results are:

$$S_\infty \overset{Exhaustible}{=} 0 \tag{54}$$

$$P_\infty = P_{\max} = P_0 + \phi S_0 \tag{55}$$

$$\hat{C}_\infty = \frac{\alpha B}{\sigma} - \frac{\alpha D\left(P_\infty\right)}{\sigma} - \frac{\rho}{\sigma} \tag{56}$$

For the final steady-state condition, we recall the transversality condition 30:

$$\lim_{t\to\infty} \lambda_{K_t} K_t e^{-\rho t} = 0 \tag{57}$$

The above expression implies that

$$\widehat{\lambda_{K_t} K_t} - \rho < 0 \tag{58}$$

Moreover, we know from Equation 45 that:

$$\widehat{\lambda_{K_t} K_t} = -B\epsilon_t + \rho \tag{59}$$

We combine the above conditions 58, 59 to get that:

$$\lim_{t\to\infty} \epsilon_t > 0 \tag{60}$$

As we know from Equation 5 that $\hat{K}_t$ is asymptotically constant for $t\to\infty$, we have that $\lim_{t\to\infty}\hat{\epsilon}_t \leq 0$. However, we know from (60) that $\hat{\epsilon}_t$ is strictly positive; hence it must be

$$\lim_{t\to\infty} \hat{\epsilon}_t = 0 \tag{61}$$

The final steady-state condition can be directly obtained from Equation 47:

$$\epsilon_\infty = \frac{1}{B}\left[(1-\sigma)\hat{C}_t - \rho\right] \tag{62}$$

As a final part of our model solution for the social optimum, we will try to derive the social cost of carbon (SCC). The SCC basically reflects total damages from releasing greenhouse gas emissions to the atmosphere at every point in time. Labeling the SCC as χ_t, that is the marginal externality damage from burning an additional unit of polluting non-renewable resource, we can define it as:

$$\chi_t = -\phi\frac{\lambda_{P_t}}{\lambda_{C_t}} \tag{63}$$

Let $(\nu - t)$ the time interval between a generic period of emission ν and the reference period of emission t. We suppose the pollution stock increases over time. Since we took ν such that $\nu > t$, we have

$$P_\nu \geq P_t \quad for\ all \quad \nu \geq t \tag{64}$$

It is then possible to rewrite the transversality condition for the stock pollution (31):

$$0 = \lim_{\nu\to\infty} \lambda_{P_\nu} P_\nu e^{-\rho\nu} \geq \lim_{\nu\to\infty} \lambda_{P_\nu} P_t e^{-\rho\nu} \tag{65}$$

which implies

$$\lim_{\nu\to\infty} \lambda_{P_\nu} e^{-\rho(\nu-t)} = 0 \quad for\ all \quad \nu \geq t \tag{66}$$

We recall Equation 28 in terms of ν:

$$\hat{\lambda}_{P_\nu} = \dot{D}(P_\nu)K_\nu\frac{\lambda_{K_\nu}}{\lambda_{P_\nu}} + \rho \xrightarrow[e^{-\rho(\nu-t)}]{Multiplying\ both\ sides\ with} \hat{\lambda}_{P_\nu}e^{-\rho(\nu-t)} = \dot{D}(P_\nu)K_\nu\frac{\lambda_{K_\nu}}{\lambda_{P_\nu}}e^{-\rho(\nu-t)} + \rho e^{-\rho(\nu-t)}; \tag{67}$$

$$\xrightarrow{Multiplying\ both\ sides\ with\ \lambda_{P_\nu}} \quad \dot{\lambda}_{P_\nu}e^{-\rho(\nu-t)} - \rho\lambda_{P_\nu}e^{-\rho(\nu-t)} = \dot{D}\left(P_\nu\right)\lambda_{K_\nu}K_\nu e^{-\rho(\nu-t)}; \tag{68}$$

$$\frac{d\left(\lambda_{P_\nu}e^{-\rho(\nu-t)}\right)}{d\nu} = \dot{D}\left(P_\nu\right)\lambda_{K_\nu}K_\nu e^{-\rho(\nu-t)}; \tag{69}$$

Using the transversality implication (66), we can calculate the indefinite integral from $\nu = t$ to $\nu \to \infty$ as

$$-\lambda_{P_t} = \int_t^\infty \dot{D}\left(P_\nu\right)\lambda_{K_\nu}K_\nu e^{-\rho(\nu-t)}d\nu \tag{70}$$

Substituting $-\lambda_{P_t}$ from (70) into (63) we get that

$$\chi_t = -\phi\frac{\lambda_{P_t}}{\lambda_{C_t}} = \frac{\phi}{\lambda_{C_t}}\int_t^\infty \dot{D}\left(P_\nu\right)\lambda_{K_\nu}K_\nu e^{-\rho(\nu-t)}d\nu \tag{71}$$

Substituting λ_{C_t} from (14) and $(\lambda_{K_\nu}K_\nu)$ from (15) we get that

$$\chi_t = \frac{\phi}{C_t^{-\sigma}}\int_t^\infty \dot{D}\left(P_\nu\right)\frac{\alpha C_\nu\lambda_{C_\nu}}{B\epsilon_\nu}e^{-\rho(\nu-t)}d\nu \tag{72}$$

We multiply and divide the right-hand side of Equation 72 with C_t and ρ:

$$\chi_t = C_t\frac{\alpha\phi}{\rho}\int_t^\infty \dot{D}\left(P_\nu\right)\left(\frac{\rho}{B\epsilon_\nu}\right)\frac{\left(C_\nu\lambda_{C_\nu}\right)}{C_t^{1-\sigma}}e^{-\rho(\nu-t)}d\nu \tag{73}$$

We substitute $\dfrac{\rho}{B}$ and λ_{C_ν} according to Equations 35 and 14, respectively.

$$SCC = \chi_t = C_t\frac{\alpha\phi}{\rho}\int_t^\infty \dot{D}\left(P_\nu\right)\left(\frac{\bar{\epsilon}}{\epsilon_\nu}\right)\left(\frac{C_t}{C_\nu}\right)^{\sigma-1}e^{-\rho(\nu-t)}d\nu \tag{74}$$

We can notice that Equation 74 is the **Social Cost of Carbon (SCC)** for the BCE model we are studying.

The first term inside the integral is the marginal damage of pollution on capital accumulation,i.e. $\dot{D}(P_t)$, the second term comes from shadow prices of capital and is responsible for allocating capital between the consumption and the investment sector, while the third term reflects preferences of agents regarding intertemporal consumption.

.3 Comparison with the decentralized case and the role of policy

In Section 2 we characterized the socially optimal solution and derived the general expression for the social cost of carbon, χ_t (see Equation 74).
Here we compare the results obtained in the model above with those for the decentralized equilibrium case. For the decentralized case, we make reference to the work done by Bretschger L. and Karydas C. in "Economics of climate change: Introducing the Basic Climate Economic (BCE) model" [2018].

After a careful look at the two models we can notice that, if expressed per units of output, the social cost of carbon (SCC) for the social planner case equals the Pigouvian carbon tax, i.e. the tax needed to optimally correct for the environmental pollution externality in the decentralized equilibrium case.
Indeed, under general conditions, we have that:

$$\widetilde{SCC} = \tilde{\chi_t} \equiv \frac{\chi_t}{Y_t} \ = \ Social\ cost\ of\ carbon\ per\ unit\ of\ output \tag{75}$$

Substituting χ_t according to Equation 74:

$$\tilde{\chi_t} \equiv -\frac{\phi\lambda_{P_t}}{\lambda_{C_t}Y_t} \quad \xrightarrow[Eqn.9]{Y_t=C_T} \quad \tilde{\chi_t} = \frac{\alpha\phi}{\rho}\int_t^{\infty}\dot{D}\left(P_\nu\right)\left(\frac{\bar{\epsilon}}{\epsilon_\nu}\right)\left(\frac{C_t}{C_\nu}\right)^{\sigma-1}e^{-\rho(\nu-t)}d\nu \tag{76}$$

and if we assume $\sigma = 1$, i.e. logarithmic utility function, we have from (32) that

$$\lambda_{C_t}C_t = 1 \Rightarrow \tilde{\chi_t} \equiv -\phi\lambda_{P_t} \quad which\ implies \quad \hat{\tilde{\chi_t}} \equiv -\phi\hat{\lambda}_{P_t} \tag{77}$$

It can be shown that in the decentralized case, given $\sigma = 1$, the capital share ϵ_t immediately jumps to its optimal steady state value $\bar{\epsilon} \equiv \rho/B$. This is the same result that we obtained with the social planner in Equation 35. We can compare the social planner's optimality condition in Equation 17 with its equivalent from the market case[1]:

Social Planner	Decentralized Equilibrium
$(1-\alpha)\frac{C_t}{R_t} = \frac{\lambda_{S_t}}{\lambda_{C_t}} - \phi\frac{\lambda_{P_t}}{\lambda_{C_t}}$	$(1-\alpha)\frac{Y_t}{R_t} = p_{R_t} + \tau_t$

Assuming the equilibrium condition on consumer goods market, i.e. $C_t = Y_t$, it is immediate to notice that the resource extraction will follow its optimal path ONLY if the producer's price for the non-renewable resource p_{R_t} equals its scarcity rent ($p_{R_t} = \lambda_{S_t}/\lambda_{C_t}$), and if the carbon tax τ_t per-unit in the decentralized case equals the marginal externality damage of emissions $-\phi\frac{\lambda_{P_t}}{\lambda_{C_t}}$ in the social planner case.
Recalling the definition of the social cost of carbon χ_t, we have

$$\tau_t = -\phi\frac{\lambda_{P_t}}{\lambda_{C_t}} = \chi_t \quad \xrightarrow[Y_t=C_t]{\widetilde{\chi_t}Y_t\equiv\chi_t} \quad \boldsymbol{\tau_t = \widetilde{\chi_t}C_t} \tag{78}$$

We have verified in Equation 78 that the social cost of carbon (SCC), derived from the social optimum model, equals the optimal per-unit carbon tax, here τ_t^0.

As long as we are considering only polluting non-renewable resources, we can see from (78) that the optimal carbon tax τ_t is proportional to the good consumption C_t for $\sigma = 1$, or it asymptotically becomes so in the long run for $\sigma \neq 1$. [1,2]
Hence a carbon tax as τ_t affects the starting point and the transition of all the control variables, but not the long-run steady state of the economy. Resource taxation delays extraction and stretches the depletion of the resource stock to the future. Comparing the baseline no-tax model with the taxation model, we can therefore suppose that during the transition phase we will have

[1] Lucas Bretschger and Christos Karydas. "Optimum growth and carbon policies with lags in the climate system". In: Environmental and Resource Economics 70.4 (2018-08), pp. 781–806. ISSN: 0924-6460. DOI: 10.1007/s10640-017-0153-4.

[2] Lucas Bretschger and Christos Karydas. "Economics of Climate Change: Introducing the Basic Climate Economic (BCE) Model". In: Environment and development economics 24.6 (2019-12), pp. 560–582. ISSN: 1355-770X. DOI: 10.3929/ethz-b-000394747.

No-Tax		Tax
$S_{baseline}$	$<$	S_{tax}
$D_{baseline}$	$>$	D_{tax}
$P_{baseline}$	$>$	P_{tax}
$\hat{C}_{baseline}$	$<$	$\hat{C}_{tax}$

Moreover, it is also logical to think that the per-unit tax that should in theory postpone extraction has to grow at a slower rate than the price of the non-renewable resource. In this way, the unit-price paid for the resource by consumers increases less rapidly than the price received by producers (which will grow at the market's interest rate ρ), encouraging them to postpone extraction.
This hypothesis is confirmed by looking at the results (79), (80) of the households optimization problem, as reported by Bretschger and Karydas (2019)[2]:

$$\hat{C}_t = \frac{1}{\sigma}(r_t - \rho) \tag{79}$$

$$\hat{p}_{R_t} = r_t \tag{80}$$

if we assume $\sigma = 1$, the price received by producers p_{R_t} grows at a rate r_t, while the optimal carbon tax τ_t^0 grows at $r_t - \rho$, (i.e. with consumption).

We can finally conclude the models comparison by stating that, given an economy in the social optimum, the economy can still have positive growth with climate change. For the decentralized equilibrium, positive growth is feasible provided that efficient climate policies, such as carbon taxes, replicating the social optimum are implemented.

Social Planner	Decentralized Equilibrium
Less consumption	A <u>TAX is needed</u> to increase the price of polluting activities
$\Rightarrow$ Less pollution	$\Rightarrow$ Decrease in the number of polluting activities

If there are <u>no</u> externalities, the results of the two models will coincide.

	Value	
σ	1.8	
ρ	0.015	
α	0.9	
δ_0	0.05	
δ_1	0.04	
δ_2	5×10^{-9}	
η	2.35	
P_0	830	GtC
S_0	6000	GtC
ϕ	1	
B	0.106	

Table 1: Parameters values for the model

.4 Baseline model simulation

Here we will try to show graphically the outcome of the simulation for the BCE social optimum baseline model. In particular, we will try to plot via MATLAB the trend with respect to time of the *Resource stock* S_t, the *Capital depreciation* D_t and the *Consumption growth rate* $\hat{C}_t$.

For the calibration of the BCE baseline model, we will use the parameters suggested by Bretschger and Karydas (2019)[3]. Despite being 2010 the initial time ($t = 0$) considered in the previous study, we will choose parameters on the damage function such that the growth rate of consumption starts at about 2 percent per annum converging to about 0.5 percent per annum in the long run. The parameters values are reported in Table 1.

As suggested in this study, we will assume that pollution feeds back in the economy through a sigmoidal damage function $D(P_t)$, according to:

$$D\left(P_t\right) = \delta_0 + \delta_1 \left(1 - \frac{1}{1 + \delta_2 \left(P_t - P_0\right)^{\eta}}\right) \tag{81}$$

For the consumption growth rate $\hat{C}_t$, we make reference to the Keynes-Ramsey rule as expressed in Equation 53:

$$\hat{C}_t = \frac{\alpha B}{\sigma} - \frac{\alpha D\left(P_t\right)}{\sigma} - \frac{\rho}{\sigma} \tag{82}$$

For the resource stock, we recall the basic Equations 3 and 7:

$$\dot{S}_t = -R_t \quad s.t. \quad \int_0^{\infty} R_t dt \leq S_0 \qquad and \qquad \dot{P}_t = \phi R_t = -\phi \dot{S}_t \tag{83}$$

which combined lead to the model equation for the resource stock S_t:

$$P_t = P_0 + \phi\left(S_0 - S_t\right) \quad \xrightarrow{Rearranging} \quad S_t = S_0 + \frac{1}{\phi}\left(P_0 - P_t\right) \tag{84}$$

In order now to simulate the model, we will use the standard linearization technique via the jacobian matrix. To do this we can introduce the auxiliary variable ψ_t, i.e. the relative shadow price of the resource stock, such that:

$$\psi_t \equiv \frac{\lambda_{S_t}}{\lambda_{S_t} - \phi \lambda_{P_t}} \quad where \quad 0 < \psi_t < 1 \tag{85}$$

Recalling Equation 28:

$$\hat{\lambda}_{P_t} = \dot{D}(P_t) K_t \frac{\lambda_{K_t}}{\lambda_{P_t}} + \rho \tag{86}$$

[3] Bretschger and Karydas, “Economics of Climate Change: Introducing the Basic Climate Economic (BCE) Model”.

We can rewrite Equation 17 using Equation 26, 28 and 85 in order to get

$$\hat{\psi}_t = (1-\psi_t)\,\frac{\alpha \dot{D}\left(P_t\right)}{B\epsilon_t\tilde{\chi}_t} \tag{87}$$

as well as

$$\xrightarrow[\text{Equations 47 and 75}]{\text{Recalling}} \quad \hat{R}_t = -B\epsilon_t + \hat{\epsilon}_t - (1-\psi_t)\,\frac{\alpha \dot{D}\left(P_t\right)}{B\epsilon_t\tilde{\chi}_t} \tag{88}$$

We now define the resource depletion rate u_t as

$$u_t = \frac{R_t}{S_t} \tag{89}$$

which allows us to rewrite Equation 3 and 7 as

$$\hat{P}_t = \phi u_t \frac{S_t}{P_t} \quad \textit{and} \quad \hat{S}_t = -u_t \tag{90}$$

If we recall Equations 37, 47, 75, 87, 89, 90, and express them in terms of $u_t, \epsilon_t, \psi_t, \tilde{\chi}_t, P_t$ and S_t we are finally able to represent the model dynamics by the following system of six variables.

$$\dot{u}_t = u_t\left(-B\epsilon_t + \frac{\dot{\epsilon}_t}{\epsilon_t} - (1-\psi_t)\,\frac{\alpha \dot{D}\left(P_t\right)}{B\epsilon_t\tilde{\chi}_t} + u_t\right) \tag{91}$$

$$\dot{\epsilon}_t = \epsilon_t\left(-\rho + B\epsilon_t - (\sigma-1)\,\hat{Y}_t\right) \tag{92}$$

$$\dot{\psi}_t = \psi_t\left((1-\psi_t)\,\frac{\alpha \dot{D}\left(P_t\right)}{B\epsilon_t\tilde{\chi}_t}\right) \tag{93}$$

$$\dot{\widetilde{\chi}}_t = \widetilde{\chi}_t\left(B\epsilon_t - \frac{\dot{\epsilon}_t}{\epsilon_t} - \frac{\alpha \dot{D}\left(P_t\right)}{B\epsilon_t\tilde{\chi}_t}\right) \tag{94}$$

$$\dot{P}_t = P_t\left(\phi u_t \frac{S_t}{P_t}\right) \tag{95}$$

$$\dot{S}_t = S_t\left(-u_t\right) \tag{96}$$

where

$$\hat{Y}_t = \alpha\left(\frac{\dot{\epsilon}_t}{\epsilon_t} + B\left(1-\epsilon_t\right) - D\left(P_t\right)\right) + (1-\alpha)\left(\frac{\dot{u}_t}{u_t} - u_t\right) \tag{97}$$

Recalling (54), (92) and (62), we write the long-run steady state values of variables $u_t, \epsilon_t, \psi_t, \tilde{\chi}_t, P_t$ and S_t:

$$u_\infty = B\epsilon_\infty \tag{98}$$

$$\epsilon_\infty = \frac{\rho + \alpha(\sigma-1)\left(B - D\left(P_\infty\right)\right)}{B\sigma} \tag{99}$$

$$\psi_\infty = 1 \tag{100}$$

$$\widetilde{\chi_\infty} = \frac{\alpha \dot{D}\left(P_\infty\right)}{\left(B\epsilon_\infty\right)^2} \tag{101}$$

$$P_\infty = P_0 + \phi S_0 \tag{102}$$

$$S_\infty = 0 \tag{103}$$

The linearized version of our dynamic system in $\boldsymbol{x_t} = \{u_t, \epsilon_t, \psi_t, \tilde{\chi_t}, P_t, S_t\}^T$ can be obtained by using the jacobian matrix $\boldsymbol{J}$ evaluated at the steady states $\boldsymbol{x_\infty} = \{u_\infty, \epsilon_\infty, \psi_\infty, \tilde{\chi_\infty}, P_\infty, S_\infty\}^T$ according to the relation:

$$\frac{d\left(\boldsymbol{x_t} - \boldsymbol{x_\infty}\right)}{dt} \approx \boldsymbol{J}\left(\boldsymbol{x_t} - \boldsymbol{x_\infty}\right) \tag{104}$$

If we compute the jacobian matrix for the steady states we obtain the eigenvalues:

$$\begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\xi & 0 & 0 & 0 & 0 \\ 0 & 0 & -\xi & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi & 0 & 0 \\ 0 & 0 & 0 & 0 & \xi & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi \end{vmatrix} \tag{105}$$

where

$$\xi = \frac{\rho + \alpha\left(\sigma - 1\right)\left(B - \eta D\left(P_\infty\right)\right)}{\sigma} \tag{106}$$

We can now exploit the approximated relation expressed in Equation 104 and the jacobian matrix 105 to derive the model equation for the resource stock S_t:

$$\frac{d\left(S_t - S_\infty\right)}{dt} \approx J_{S_t S_t}\left(S_t - S_\infty\right) \quad \xrightarrow[J_{S_t S_t} = -\xi]{Eqn.103} \quad \frac{d\left(S_t\right)}{dt} \approx -\xi S_t \tag{107}$$

$$\dot{S}_t \approx -\xi S_t \tag{108}$$

Equation 108 can be solved both analytically and numerically. Here we propose the analytical approach first:

$$\xrightarrow{Solving\ the\ ODE} \quad S_t \approx C_1 e^{-\xi t} \quad we\ know\ that\ S(t=0) = S_0 \quad \Rightarrow \quad C_1 = S_0 \tag{109}$$

therefore

$$S_t \approx S_0 e^{-\xi t} \quad \rightarrow \quad S_t \approx S_0 e^{-\dfrac{\rho + \alpha\left(\sigma - 1\right)\left(B - \eta D\left(P_\infty\right)\right)}{\sigma} t}; \tag{110}$$

$$S_t \approx S_0 e^{-\dfrac{\rho + \alpha\left(\sigma - 1\right)\left(B - \eta\left(\delta_0 + \delta_1\left(1 - \dfrac{1}{1 + \delta_2\left(\left(P_0 + \phi S_o\right) - P_0\right)^\eta}\right)\right)\right)}{\sigma} t} \tag{111}$$

We have found the expression for the time evolution of the resource stock S_t, which will be a function of time exclusively, since all the parameters in Equation 111 are known.

However, we will plot the time-trends of the *Resource stock* S_t, the *Capital depreciation* D_t and the *Consumption growth rate* $\hat{C}_t$ numerically using MATLAB.
In order to solve numerically the equation 108, we will use the Runge-Kutta method for numerical differentiation. We will assume a time period of interest of 200 years. Hereunder follows the MATLAB script for the numerical analysis.

.5 MATLAB Code

```
clc; clear all; close all;
%% PARAMETERS SETUP %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Parameters values from Table 1

sigma = 1;
rho = 0.015;
alpha = 0.9;
delta_0 = 0.05;
delta_1 = 0.04;
delta_2 = 5*10^(-9);
eta = 2.35;
P_0 = 830;
S_0 = 6000;
phi = 1;
B = 0.106;

P_t=0; S_t=0;     %Random initial values
% Sigmoidal damage function D[t]
D=delta_0+delta_1*(1-(1)/(1+delta_2*(P_t-P_0)^(eta)));
% Long-run steady state value for the stock Pollution P_t
P_inf=P_0+phi*S_0;

% We derived in Equation 108 the ODE for the stock resource S_t:
S_t_prime = -((rho+alpha*(sigma-1)*(B-eta*D))/(sigma))*S_t;

%% 4th-order RUNGE-KUTTA NUMERICAL DIFFERENTIATION %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% We will solve the ODE relative to S_t using the Runge-Kutta 4th-order method.
% Here we assume for simplicity y=S_t

% Equation to solve: Y'=-((rho+alpha*(sigma-1)*(B-eta*D(P_inf)))/(sigma))*Y;
%                    Y(0)=S_0; t=[0,200];

fid=fopen('Runge-Kutta_increments.m','w');  % Write results on an external file

h=0.1; a=0; b=200;            % h is the step size, t=[a,b] t-range
t = a:h:b;                    % Computes t-array
y = zeros(1,numel(t));        % Memory preallocation

P_t=P_inf;
y(1) = S_0;                          % initial condition; in MATLAB indices start at 1
Fyt = @(t,y) -((rho+alpha*(sigma-1)*(B-eta*D))/(sigma))*y;    % The function is the
                                                              % expression after (t,y)

% Table title
fprintf(fid,'%7s %7s %7s %7s %7s %7s %7s \n','i','t(i)','k1','k2','k3', 'k4','y(i)');
for ii=1:1:numel(t)
    k1 = Fyt(t(ii),y(ii));
    k2 = Fyt(t(ii)+0.5*h,y(ii)+0.5*h*k1);
    k3 = Fyt((t(ii)+0.5*h),(y(ii)+0.5*h*k2)); k4 = Fyt((t(ii)+h),(y(ii)+h*k3));
    y(ii+1) = y(ii) + (h/6)*(k1+2*k2+2*k3+k4);      % Main equation
```

```
    % Table data
    fprintf(fid,'%7d %7.2f %7.3f %7.3f',ii, t(ii), k1, k2);
    fprintf(fid,' %7.3f %7.3f %7.3f \n', k3, k4, y(ii));
end
y(numel(t))=[ ];          % Erase the last computation of y(n+1)

% Solution PLOT:
figure(1);
f1=plot(t,y,'DisplayName','S_t','MarkerSize',3,'Marker','o','LineStyle','none','Color',
   [0 0 0]); hold all;
title('Time evolution of the Resource Stock S_t','FontSize',14);
ylabel('Resource stock S_t'); xlabel('Time [years]');
box('on'); set(gca,'XMinorTick','on','YMinorTick','on');

S_t=get(f1,'YData');     % YData extrapolation from 'figure 1'
fclose(fid);

%% Derivation and plot for the POLLUTION STOCK P_t  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% We recall Equation 84

P_t = P_0+phi*(S_0-S_t);

% Solution PLOT:
figure(2);
f2=plot(t,P_t,'DisplayName','P_t','MarkerSize',3,'Marker','o','LineStyle','none',
   'Color',[0 0 0]); hold all;
title('Time evolution of the Pollution Stock P_t','FontSize',14);
ylabel('Pollution stock P_t'); xlabel('Time [years]');
box('on'); set(gca,'XMinorTick','on','YMinorTick','on');

P_t=get(f2,'YData');     % YData extrapolation from 'figure 2'

%% Derivation and plot for the sigmoid DAMAGE FUNCTION D(P_t)  %%%%%%%%%%%%%%%%%%%%%%
% We recall Equation 81

D_t=delta_0+delta_1*(1-(1)./(1+delta_2*(P_t-P_0).^(eta)));

% Solution PLOT:
figure(3);
f3=plot(t,D_t,'DisplayName','D_t)','MarkerSize',3,'Marker','o','LineStyle','none',
   'Color',[0 0 0]); hold all;
title('Time evolution of the Capital Depreciation D(P_t)','FontSize',14);
ylabel('Damage Function D(P_t)'); xlabel('Time [years]');
box('on'); set(gca,'XMinorTick','on','YMinorTick','on');

D_t=get(f3,'YData');     % YData extrapolation from 'figure 3'

%% Derivation and plot for the CONSUMPTION GROWTH RATE C^_t  %%%%%%%%%%%%%%%%%%%%%%%%
% We recall Equation 82

C_t = (1/sigma )*(alpha*B-alpha.*D_t-rho);

% Solution PLOT:
```

```
figure(4);
f4=plot(t,C_t,'DisplayName','C_t)','MarkerSize',3,'Marker','o','LineStyle','none',
   'Color',[0 0 0]); hold all;
title('Time evolution of the Consumption Growth Rate C_t','FontSize',14);
ylabel('Consumption Growth Rate C_t'); xlabel('Time [years]');
box('on'); set(gca,'XMinorTick','on','YMinorTick','on');

C_t=get(f4,'YData');     % YData extrapolation from 'figure 4'
```

.6 Graphical results

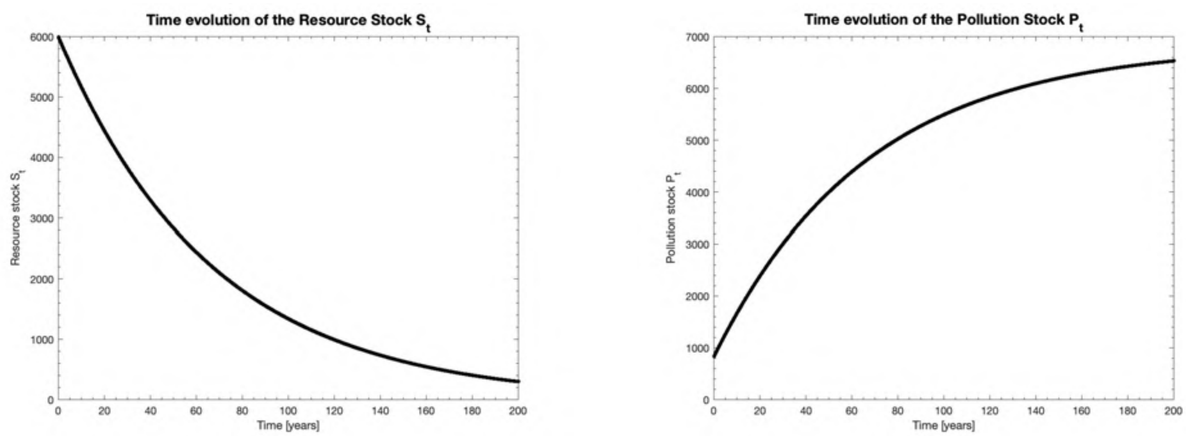

Figure 1: Resource stock S_t (on the left) and Pollution stock P_t (on the right) as a function of time t.

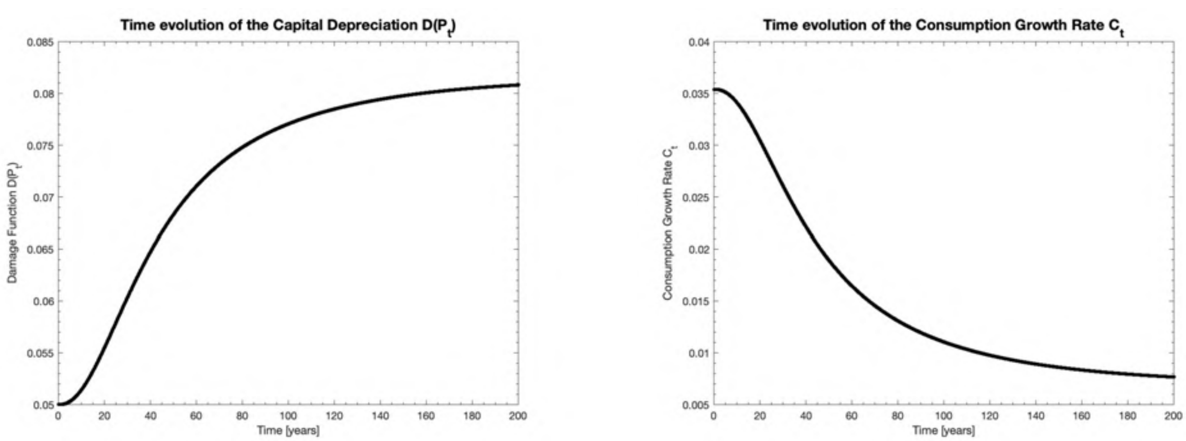

Figure 2: Capital depreciation $D(P_t)$ (on the left) and Consumption growth rate $\hat{C}_t$ (on the right) as a function of time t.

The role of population growth in global sustainable development

Population growth is often seen as a threat to sustainable development...is it true?

World population is currently growing at the highest rate ever attained in history. Thomas Malthus in the late 18th century first introduced the "Malthusian trap", suggesting that there is a limit to population size and growth due to a finite natural resource base[4]. In a similar way, Paul Ehrlich (1994)[5] also claimed that "sustainable development cannot continue without limits to population growth", referring to capital accumulation and technical progress as the principal factors that could help overcome resource scarcity and food shortages. However, the pessimistic Malthusian notion of future "misery and vice" may not be fully appropriate. Despite the evident scarcity of energy resources, raw materials deposits and the atmospheric capacity to absorb polluting emissions, the total use of natural resources and energies will ultimately have to shrink in future centuries. The question therefore remains whether policy should actively limit population growth, even in the name of sustainable development.

From a mechanistic point of view, if we look at the so-called "IPAT" formula, per capita use of natural resources and limiting population are interchangeable. Given that affluence (A) is measured by income per capita (Y/P) and technology (T) is measured by resource use per income (R/Y), human impact on nature I equals resource use R, since income Y and population P cancel in the IPAT equation. This result, despite being always true, does not contribute to our understanding of the relation between population and sustainability. Indeed, the IPAT formula is based on the assumption that population, affluence, and technology are independent of one other. In reality, these variables are highly interdependent in many ways: we can have effects such as John Hicks' "induced" innovation and the so-called "demographic transition"[6], which deals with the impact of income and wealth on fertility. On average, wealthier countries tend to have smaller families than less developed countries; this means that both public social security and the increasing costs of child parenting are among the drivers of the transition.

The relationship between the impact of population growth and the capital accumulation is also a main issue. In the traditional neoclassical view, population growth is not favorable for development. By taking into account physical capital (i.e. machines and infrastructures) only, different stocks have to be shared among a rising number of people. In other words, the use of the capital by one person affects the use by another person, consequently reducing the capital per workplace, together with labor productivity and growth. To be realistic, however, basic parts of capital come also in the form of knowledge capital, which can be shared by everybody, as well as by an increasing workforce. If new ideas arise, everyone can use them without harming the knowledge of somebody else. If we consider knowledge capital, then population growth will not decrease labor productivity. In addition, as suggested by D. Gale Johnson (2001)[7], people specialized in the creation of knowledge work in research institutes and universities, which are clearly labor-intensive. These institutions can discover and promote substitutes, including clean goods and green technologies, which will reduce the overall natural resource consumption. As Julian Simon wrote in 1981[8], population could be the "ultimate solution" to resource scarcities and environmental problems, since people can innovate. More people generate more ideas, making the education and the size of the labor force augment the intensity of knowledge creation and consequently the economic growth rate.

However, it is now worth asking if these new elements are influential enough to change the general opinion of population growth and resource scarcity. A possible answer can be found in the paper "Population Growth and Natural Resource Scarcity: Long-Run Development under Seemingly Unfavorable Conditions"[9], where it was proofed under very restrictive assumptions (i.e. poor input substitution, increasing resource prices

[4] Thomas Robert Malthus. "An essay on the principle of population. 1798". In: *The Works of Thomas Robert Malthus, London, Pickering & Chatto Publishers* 1 (1986), pp. 1–139.

[5] Joseph H. Vogel. "The Population Explosion by Paul R. Ehrlich and Anne H. Ehrlich (Simon and Schuster, NewYork, 1990), pp. 320, $US18.95, ISBN 0-671-68984-3". In: *Prometheus* 9.2 (1991), pp. 396–397. DOI: 10.1080/08109029108631961. eprint: https://doi.org/10.1080/08109029108631961. URL: https://doi.org/10.1080/08109029108631961.

[6] John R Hicks. "TheTheoryofWages". In: *London: Macmiilan* (1932).

[7] D Gale Johnson. "On population and resources: a comment". In: *Population and Development Review* 27.4 (2001), pp. 739–747.

[8] Simon JL. *The Ultimate Resource*. 1981.

[9] Lucas Bretschger. "Population Growth and Natural-Resource Scarcity: Long-Run Development under Seemingly Unfavorable Conditions". In: *The Scandinavian Journal of Economics* 115.3 (2013), pp. 722–755.

and the decision of families to have children) that population growth is not only positive, but even needed to ensure sufficient innovation. This is because it may help the economy during the transition phase by increasing the chances of developing efficient technologies. The above conclusion is also in conformity with Esther Boserup's research[10] on poor agrarian societies, where it is claimed that "necessity is the mother of invention". As she said in the book "The Conditions of Agricultural Growth: The Economics of Agrarian Change under Population Pressure", only in times of pressure people find out ways to increase the productivity by fundamentally innovating.

Moreover, in contrast to the idea that population size is a global concern, one can show that a growing labor force can be compatible with the natural environment, provided that the increasing resource scarcity is fully reflected in resource prices. To express the concept into the right perspective, the current per capita use of global resources is comparatively low in countries with high population growth, while it is much higher in rich countries. Instead of restricting population in developing countries, it could be proposed the population size to be restricted in richer countries. In many cases, however, the so-called "population problem" is actually a problem strongly related to the attitude of the individuals: to ensure sustainability, we need to constantly reduce the use of natural resource and to provide sufficiently high technical change. A transition to a long-run steady state with constant population, sustainable resource use, and positive consumption growth may be reached via a demographic transition which relies on individual behavior.

Finally, instead of a population policy which may result counterproductive, encouraging innovation, raising the prices of natural resources, and increasing living standards may induce the demographic transition and simultaneously promote sustainable consumption. As long as we don't have a shrinking population problem, just like in Japan, raising resource prices together with facilitating labor reallocation from knowledge-extensive to knowledge-intensive sectors, which can develop and exploit green technologies, are the best means to support sustainable development. Indeed, as shown in the paper "Population Growth and Natural Resource Scarcity"[11] adjusting resource process via continuous small steps to a sustainable equilibrium will help the economy. As soon as all the countries have achieved an average decent living standard, population growth is expected to stop.

[10] David Grigg. "Ester Boserup's theory of agrarian change: a critical review". In: *Progress in Geography* 3.1 (1979), pp. 64–84.

[11] Bretschger, "Population Growth and Natural-Resource Scarcity: Long-Run Development under Seemingly Unfavorable Conditions".

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— "Optimum growth and carbon policies with lags in the climate system". In: Environmental and Resource Economics 70.4 (2018-08), pp. 781–806. ISSN: 0924-6460. DOI: 10.1007/s10640-017-0153-4.

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Malthus, Thomas Robert. "An essay on the principle of population. 1798". In: *The Works of Thomas Robert Malthus, London, Pickering & Chatto Publishers* 1 (1986), pp. 1–139.

Vogel, Joseph H. "The Population Explosion by Paul R. Ehrlich and Anne H. Ehrlich (Simon and Schuster, NewYork, 1990), pp. 320, $US18.95, ISBN 0-671-68984-3". In: *Prometheus* 9.2 (1991), pp. 396–397. DOI: 10.1080/08109029108631961. eprint: https://doi.org/10.1080/08109029108631961. URL: https://doi.org/10.1080/08109029108631961.